

Amendments to the Specification:

Please amend paragraph 94 to correct the symbols as follows:

[0094] $[\equiv] \delta_k(x) = 1$ if x is closest to m_k , otherwise $[\equiv] \delta_k(x) = 0$ (resolve ties arbitrarily). The summation of these functions over a data set (see (3) and (4)) residing on the l^{th} unit gives the count, $n_{k,l}$, first moment, $[\equiv] \underline{\Sigma}_{k,l}$, and the second moment, $s_{k,l}$, of the clusters. The vector $\{n_{k,l}, [\equiv] \underline{\Sigma}_{k,l}, s_{k,l} \mid k=1, \dots, K\}$, has dimensionality $2 \cdot K + K \cdot \text{dim}$, which is the size of the SS that have to be communicated between the Integrator and each computing unit.

Please amend paragraph 95 to correct the symbols as follows:

[0095] The set of SS presented here is more than sufficient for the simple version of K-Means algorithm. The aggregated quantity, $[\equiv] \underline{\Sigma}_k s_{k,l}$, could be sent instead of the individual $s_{k,l}$. But there are other variations of K-Means performance functions that require individual $s_{k,l}$, for evaluating the performance functions. Besides, the quantities that dominate the communication cost are $[\equiv] \underline{\Sigma}_{k,l}$.

Please amend paragraph 96 to correct the symbols and the equation. In the equation, please note the distinction between the summation symbol and the subscripted variable Σ .

[0096] The l^{th} computing unit collects the SS, $\{n_{k,l}, [\equiv] \underline{\Sigma}_{k,l}, s_{k,l} \mid k=1, \dots, K\}$, on the data in its own memory, and then sends them to the Integrator. The Integrator simply adds up the SS from each unit to get the global SS,

$$\begin{aligned} n_k &= \sum_{l=1}^L n_{k,l}, \quad \Sigma_k = \sum_{l=1}^L \Sigma_{k,l}, \quad s_k = \sum_{l=1}^L s_{k,l} \\ n_k &= \sum_{l=1}^L n_{k,l}, \quad \Sigma_k = \sum_{l=1}^L \Sigma_{k,l}, \quad s_k = \sum_{l=1}^L s_{k,l}. \end{aligned}$$

Please amend paragraph 97 to correct the symbols as follows:

[0097] The leading cost of integration is $O(K \cdot \text{dim} \cdot L)$, where L is the number of computing units. The new location of the k^{th} center is given by $m_k = [\equiv] \sum_k / n_k$ from the global SS (this is the $I(\cdot)$ function in (2)), which is the only information all the computing units need to start the next iteration. The performance function is calculated by (proof by direct verification),

$$\text{Perf}_{KM} = \sum_{l=1}^L s_k.$$

Please amend paragraph 102 to correct the equation. Note that the numerator in the first summation has changed from "1" to "x".

[0102] (K-Means is similar, except its weights are the nearest-center membership functions, making its centers centroids of the cluster.) Overall then, the recursion equation is given by

$$\begin{aligned} m_k &= \frac{\sum_{x \in S} \frac{1}{d_{x,k}^3 \left(\sum_{l=1}^K \frac{1}{d_{x,l}^2} \right)^2}}{\sum_{x \in S} \frac{1}{d_{x,k}^3 \left(\sum_{l=1}^K \frac{1}{d_{x,l}^2} \right)^2}} \\ m_k &= \frac{\sum_{x \in S} \frac{x}{d_{x,k}^3 \left(\sum_{l=1}^K \frac{1}{d_{x,l}^2} \right)^2}}{\sum_{x \in S} \frac{1}{d_{x,k}^3 \left(\sum_{l=1}^K \frac{1}{d_{x,l}^2} \right)^2}}. \end{aligned} \quad (9)$$

Please amend paragraph 103 to correct the equation. Note that the index for the summation is "k" rather than "λ", and the denominators in the expression for g₂ and g₃ are cubed "3" rather than taken to the S power "S".

[0103] where $d_{x,k} = \|x - m_k\|$ and s is a constant $\cong 4$. The decomposed functions for calculating SS (see (3) and (4)) are then

$$\left\{ \begin{array}{l} g_1(x, M) = 1 / \sum_{\lambda=1}^K \frac{1}{d_{x,k}^2} \\ g_2(x, M) = g_1^2(x, M) \cdot \left(\frac{1}{d_{x,1}^S}, \frac{1}{d_{x,2}^S}, \dots, \frac{1}{d_{x,K}^S} \right) \\ g_3(x, M) = g_1^2(x, M) \cdot \left(\frac{1}{d_{x,1}^S}, \frac{1}{d_{x,2}^S}, \dots, \frac{1}{d_{x,K}^S} \right) \end{array} \right\} \left\{ \begin{array}{l} g_1(x, M) = 1 / \sum_{k=1}^K \frac{1}{d_{x,k}^2} \\ g_2(x, M) = g_1^2(x, M) \cdot \left(\frac{1}{d_{x,1}^3}, \frac{1}{d_{x,2}^3}, \dots, \frac{1}{d_{x,K}^3} \right) \\ g_3(x, M) = g_1^2(x, M) \cdot \left(\frac{x}{d_{x,1}^3}, \frac{x}{d_{x,2}^3}, \dots, \frac{x}{d_{x,K}^3} \right) \end{array} \right.$$

Please amend paragraph 108 to correct the equation. Note that the subscript for p is "k" rather than "λ", and the symbol for summation must be carefully distinguished from the subscripted variable Σ.

[0108] In this example, the EM algorithm with linear mixing of K bell-shape (Gaussian) functions is described. Unlike K-Means and K-Harmonic Means in which only the centers are to be estimated, the EM algorithm estimates the centers, the co-variance matrices, Σ_k , and the mixing probabilities, $p(m_k)$. The performance function of the EM algorithm is

$$\text{Perf}_{EM}(X, M, \Sigma, p) = -\log \left\{ \prod_{x \in S} \left[\sum_{k=1}^K p_k \cdot \frac{1}{\sqrt{(2\pi)^D \det(\Sigma_k)}} \cdot \exp \left(-\frac{1}{2} (x - m_k)^T \Sigma_k^{-1} (x - m_k) \right) \right] \right\}$$

$$\text{Perf}_{EM}(X, M, \Sigma, p) = -\log \left\{ \prod_{x \in S} \left[\sum_{k=1}^K p_k \cdot \frac{1}{\sqrt{(2\pi)^D \det(\Sigma_k)}} \cdot \exp \left(-\frac{1}{2} (x - m_k)^T \Sigma_k^{-1} (x - m_k) \right) \right] \right\}$$

(13)

Please amend paragraph 111 to correct the equation. Note that the symbol for summation must be carefully distinguished from the subscripted variable Σ_k .

[0111] where $p(x|m)$ is the prior probability with Gaussian distribution, and $p(m_k)$ is the mixing probability.

$$p(x | m_k) = \frac{1}{\sqrt{(2\pi)^D \det(\sum_k)}} \cdot \text{EXP} \left(- (x - m_k) \sum_k^{-1} (x - m_k)^T \right)$$

$$p(x | m_k) = \frac{1}{\sqrt{(2\pi)^D \det(\Sigma_k)}} \cdot \text{EXP} \left(- (x - m_k) \Sigma_k^{-1} (x - m_k)^T \right) \quad (15)$$

Please amend paragraph 112 to correct the equation. Note that the subscript for m is "k" rather than "λ", and the symbol for summation must be carefully distinguished from the subscripted variable Σ .

[0112] M-Step: With the fuzzy membership function from the E-Step, find the new center locations, new co-variance matrices, and new mixing probabilities that maximize the performance function.

$$m_k = \frac{\sum_{x \in S} p(m_k | x) \cdot x}{\sum_{x \in S} p(m_k | x)}, \sum_k = \frac{\sum_{x \in S} p(m_k | x) \cdot (x - m_k)^T (x - m_k)}{\sum_{x \in S} p(m_k | x)}, p(m_k) = \frac{1}{|S|} \sum_{x \in S} p(m_k | x)$$

$$m_k = \frac{\sum_{x \in S} p(m_k | x) \cdot x}{\sum_{x \in S} p(m_k | x)}, \Sigma_k = \frac{\sum_{x \in S} p(m_k | x) \cdot (x - m_k)^T (x - m_k)}{\sum_{x \in S} p(m_k | x)}, p(m_k) = \frac{1}{|S|} \sum_{x \in S} p(m_k | x). \quad (16-18)$$

Please amend paragraph 113 to correct the equation. Note that in the expression for f_1 , the subscript for m is "k" rather than " λ ".

[0113] The functions for calculating the SS are:

$$\begin{aligned} \cancel{f_1(x, M, \Sigma, p)} &= \cancel{-\log \left[\sum_{l=1}^K p(x | m_l) p(m_l) \right]} \\ f_1(x, M, \Sigma, p) &= -\log \left[\sum_{l=1}^K p(x | m_l) p(m_l) \right] \\ g_1(x, M, \Sigma, p) &= (p(m_1 | x), p(m_2 | x), \dots, p(m_K | x)) \\ g_2(x, M, \Sigma, p) &= (p(m_1 | x)x, p(m_2 | x)x, \dots, p(m_K | x)x) \\ g_3(x, M, \Sigma, p) &= (p(m_1 | x)x^T x, p(m_2 | x)x^T x, \dots, p(m_K | x)x^T x) \end{aligned}$$